# Quark and gluon contributions to the spin and momentum of the nucleon from lattice QCD


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The Cyprus Institute


European Twisted Mass Collaboration

POETIC7, 15 November 2016,
Temple University,
Philadelphia

The Cyprus Institute

CaSToRC

★Results here by Cyprus — DESY — Temple

C. Alexandrou, S. Bacchio, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, F. Steffens, A. Vaquero, C. Wiese

★ETMC collaboration

Cyprus (U. Cyprus and Cyprus Inst.)

Germany (Berlin/Zeuthen, Bonn, Frankfurt)

Italy (Rome I,II,III, Trento)

Poland (Poznan)

Switzerland (Bern)

UK (Liverpool)

# Outline

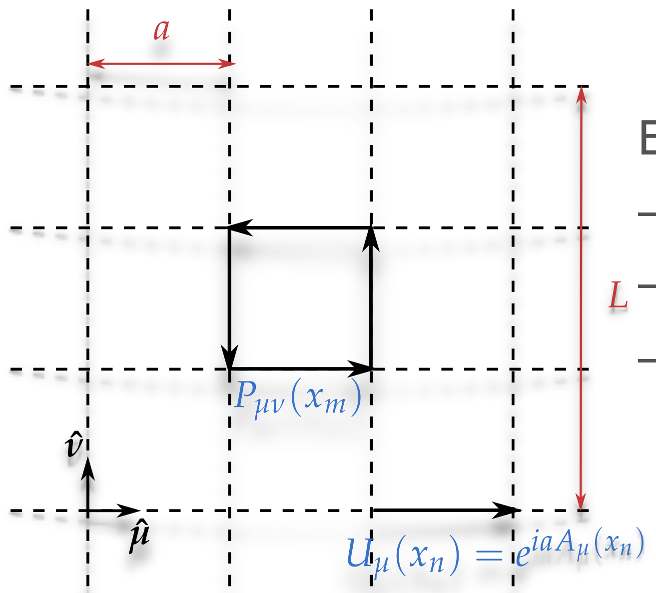
- ★ **Short introduction to lattice calculations**
  - Challenges and current landscape
- ★ **Nucleon spin on the lattice**
  - Axial charges
  - Moments of PDFs
  - Spin decomposition
  - Gluon moment
- ★ **Summary and outlook**

# Lattice QCD — *ab initio* simulation of QCD

- Freedom in choice of:
  - quark masses (*heavier is cheaper*)
  - lattice spacing *a* (*larger is cheaper*)
  - lattice volume $L^3 \times T$ (*smaller is cheaper*)
- Choice of discretisation scheme

  e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall

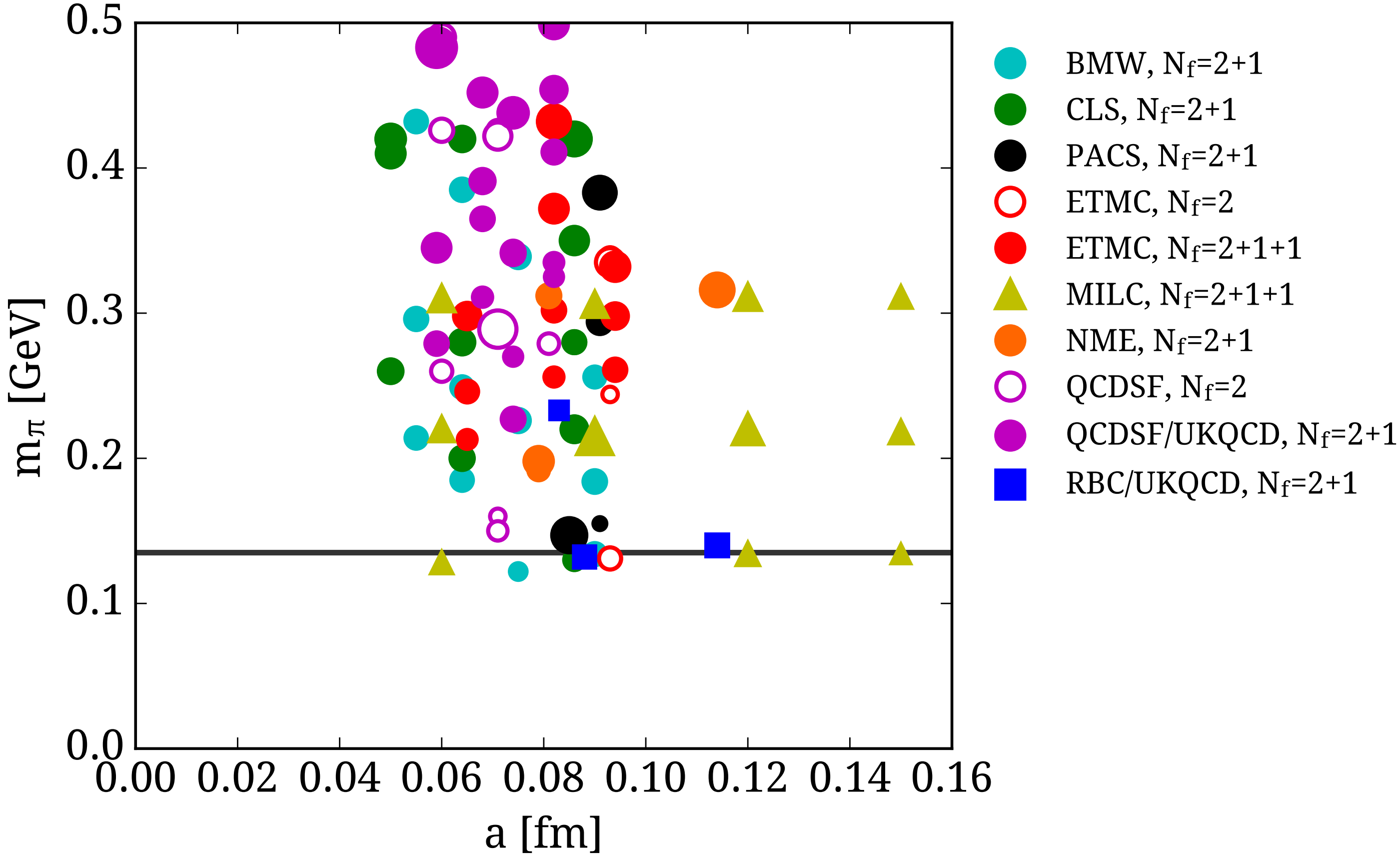
  Trade — offs and advantages for each differ




Eventually, all schemes must agree:

- At the continuum limit: $a \to 0$
- At infinite volume limit $L \to \infty$
- At physical quark mass

# Simulations landscape

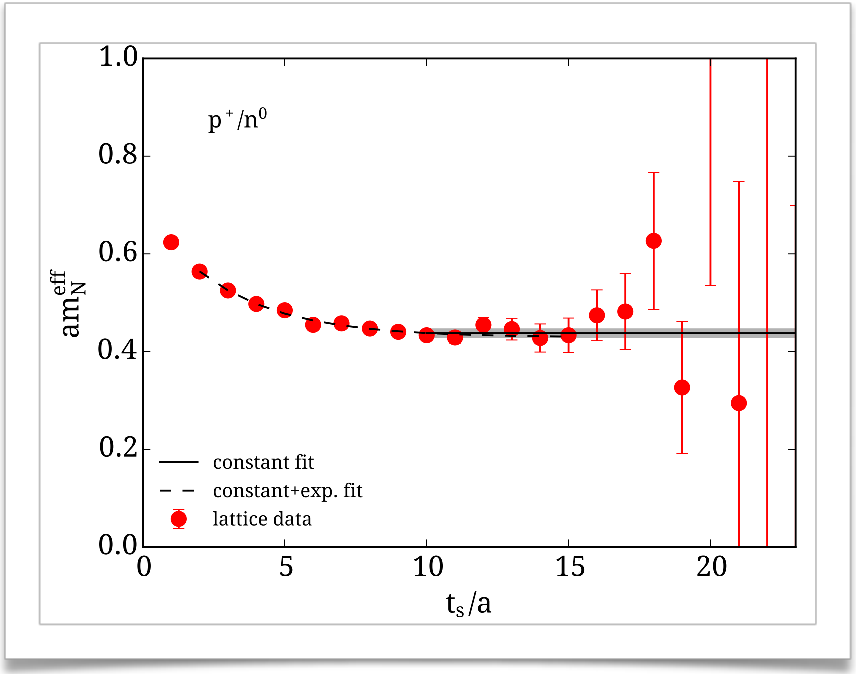


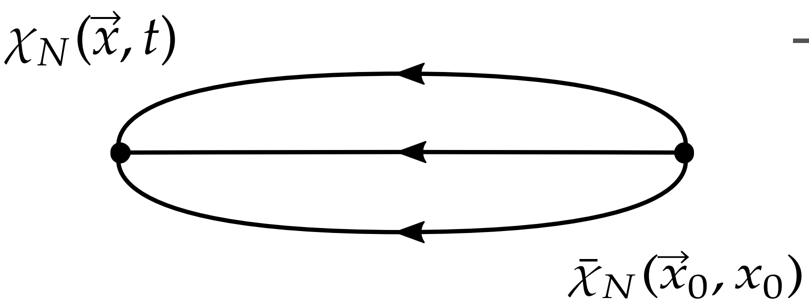

Selected lattice simulation points used for hadron structure

- Multiple collabs. simulating at physical pion mass
- Size of points indicates $m_\pi L$

## Sources of uncertainty

- Statistical error: $\frac{1}{\sqrt{N}}$, with MC samples
- Correlation functions: exponentially decay with time-separation
- Disconnected contributions: stochastic error




- Systematic uncertainties
  - Extrapolations
    $a$, $L$, $m_\pi$
  - Contamination from higher energy states

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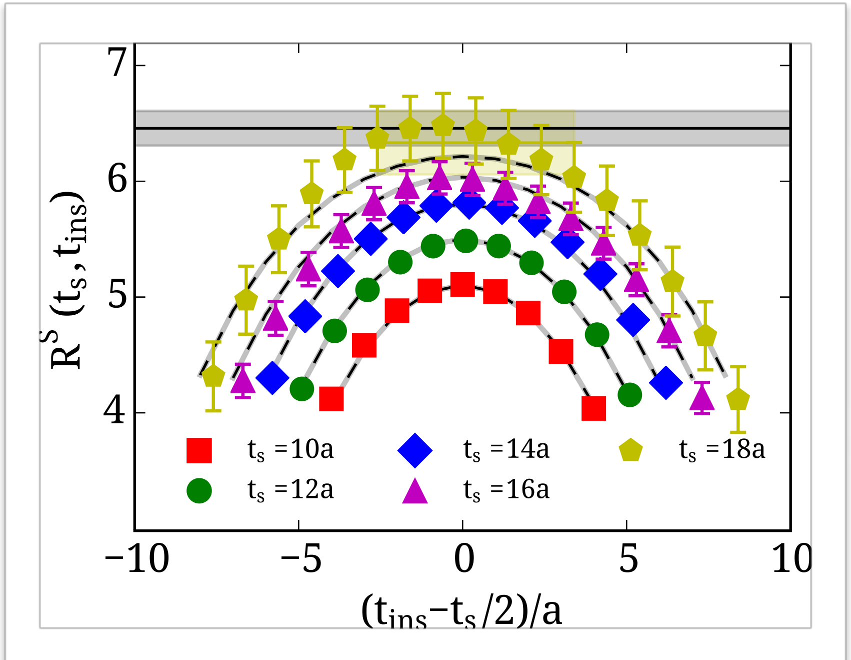


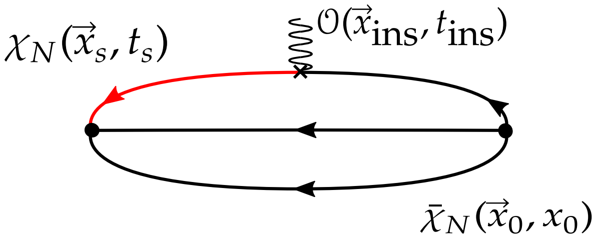


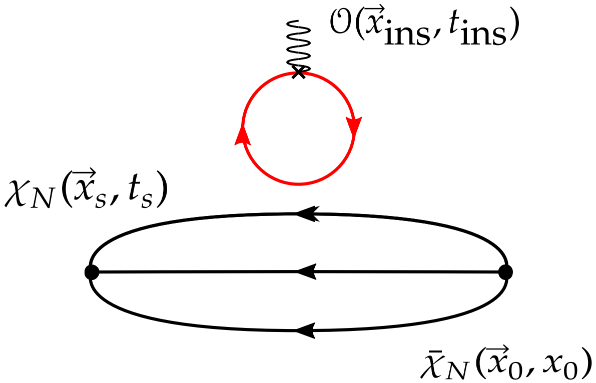


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    $a$, $L$, $m_\pi$
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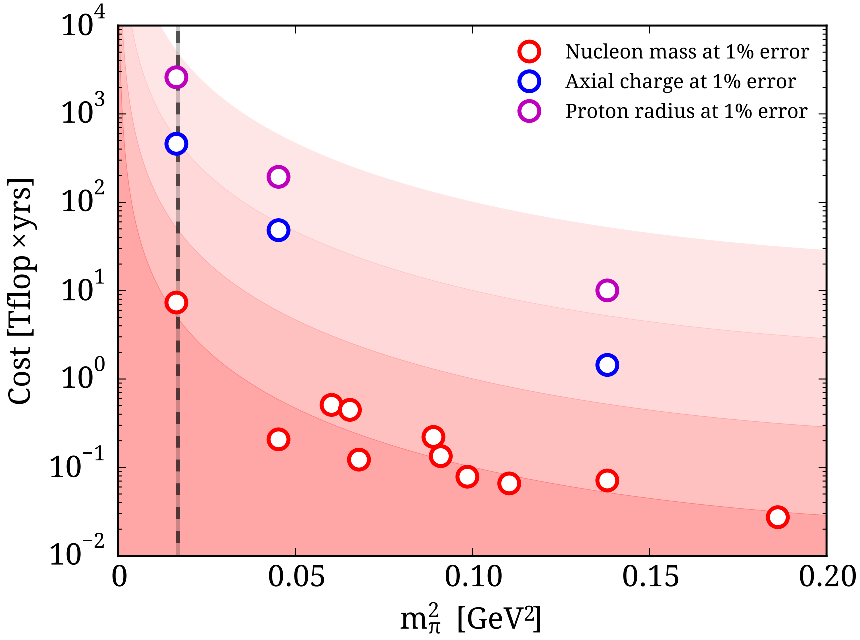
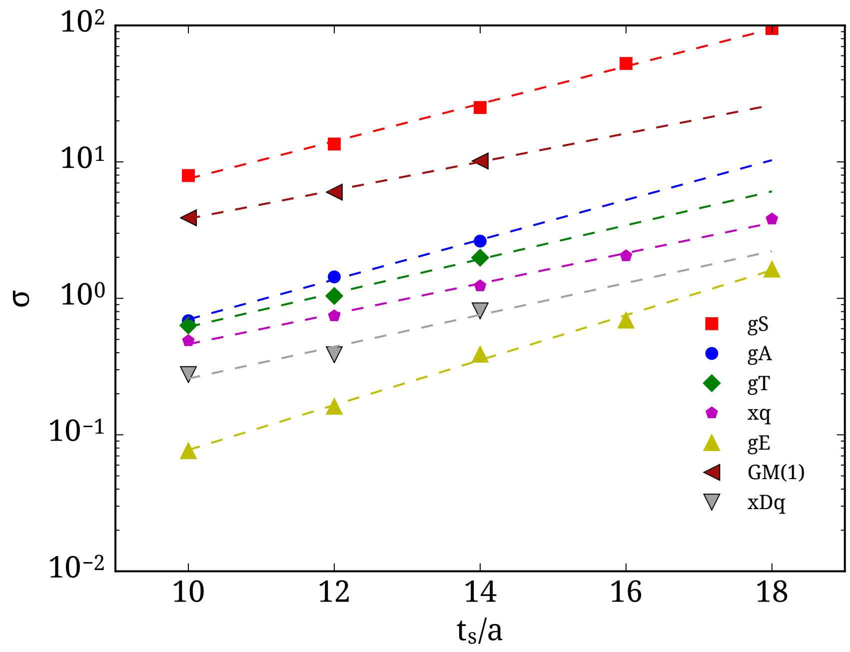
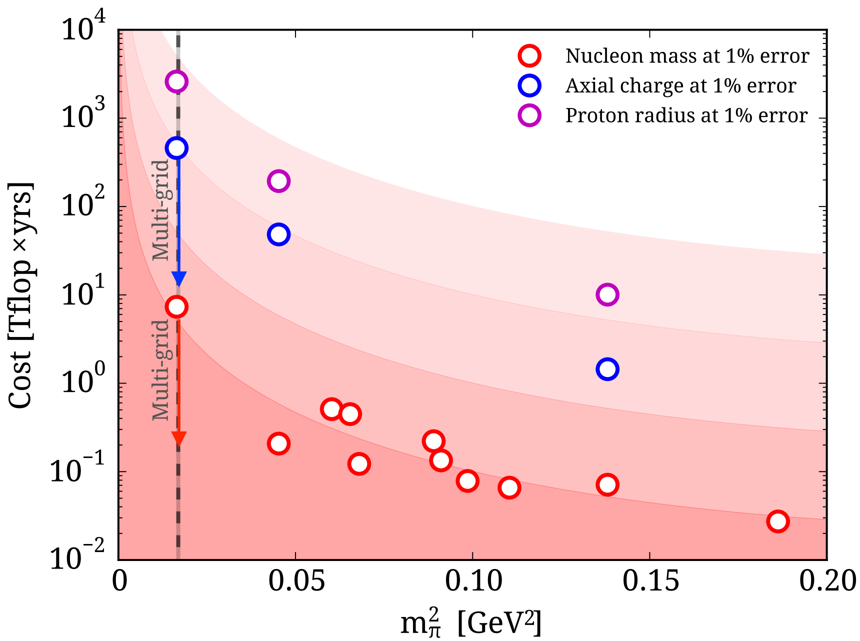
# Multi-petascale to exa-scale requirements

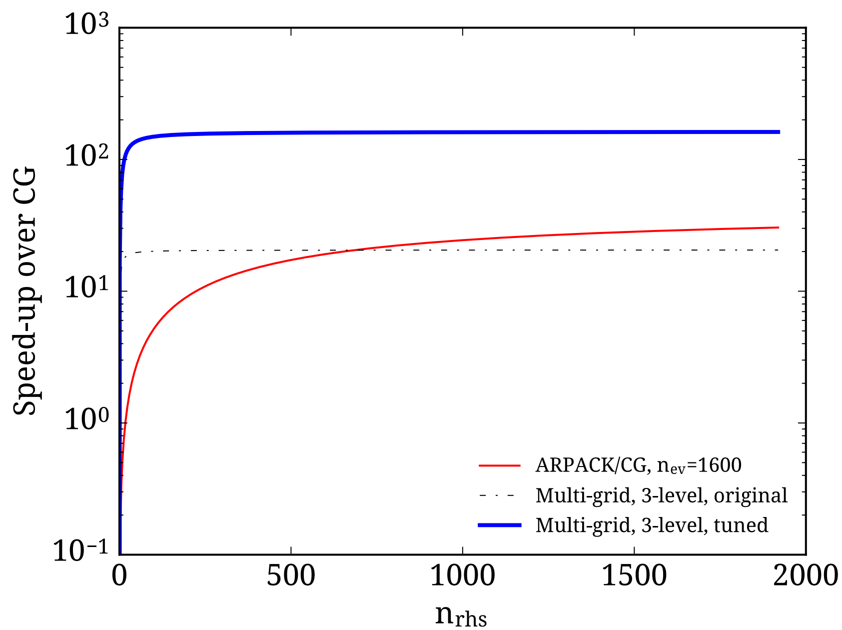


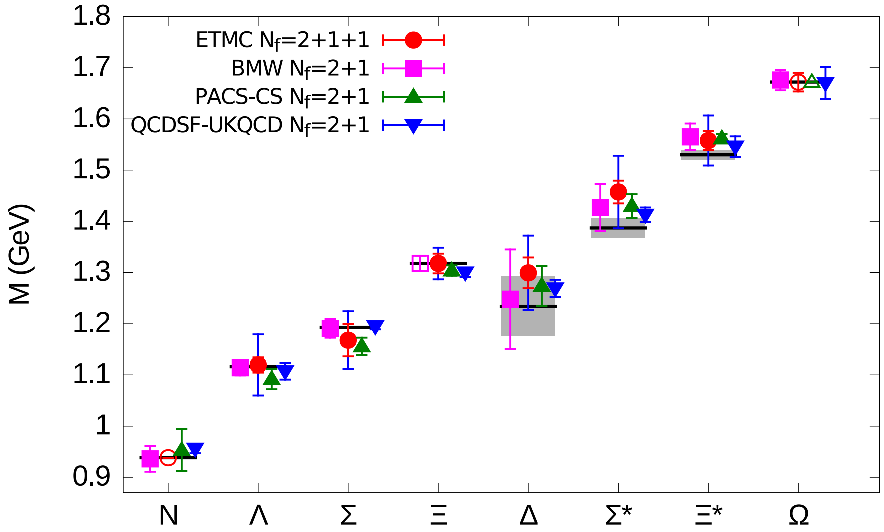

**Indicative** computer time requirements for nucleon structure ⟼ Increased time separations required for suppression of excited states at physical point

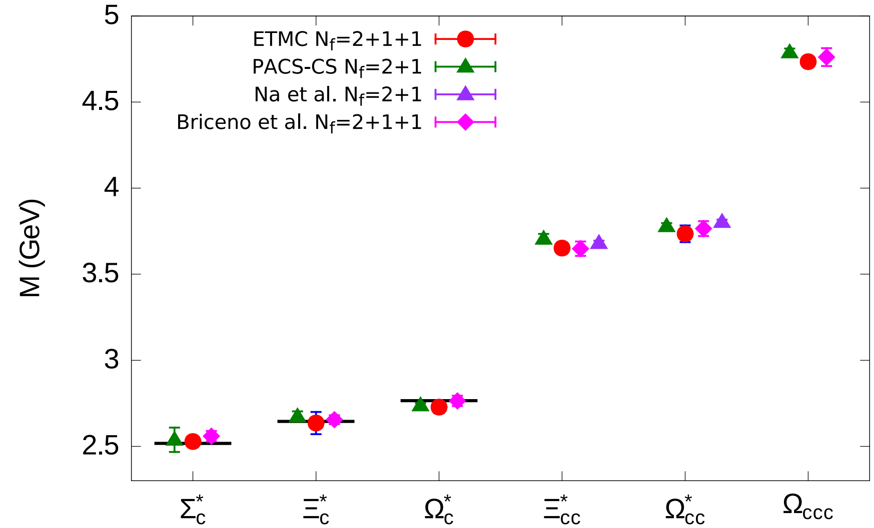
# Multi-petascale to exa-scale requirements

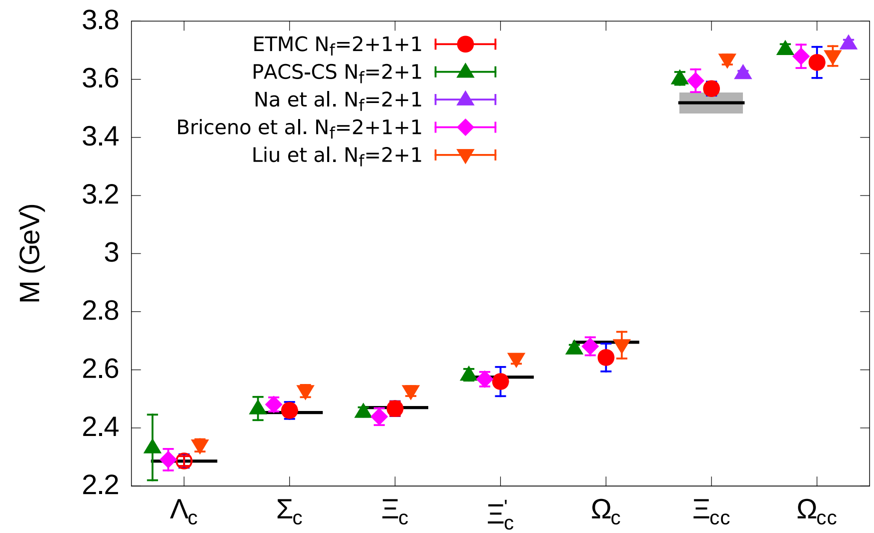



**Indicative** computer time requirements for nucleon structure $\longmapsto$ Multi-grid/deflation methods for large speed-ups, especially at physical point




Reproduction of light baryon masses

- Agreement between lattice discretisation schemes
- Reproduction of experiment

Prediction of yet-to-be-observed charmed baryons

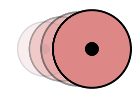
- Confidence through agreement between lattice schemes
- Nucleon structure…

# Nucleon structure on the lattice

– **Lattice:** moments of GPDs are readily accessible

## Unpolarised

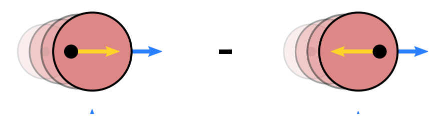
$$\mathcal{O}_{\boxed{V}}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\boxed{\gamma}^{\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$



$$\langle 1 \rangle_{u-d} = g_V\,,\quad \langle x \rangle_{u-d},\quad \ldots$$

## Helicity

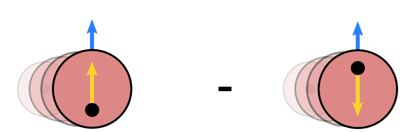
$$\mathcal{O}_{\boxed{A}}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\boxed{\gamma_5\gamma}^{\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$



$$\langle 1 \rangle_{\Delta u-\Delta d} = g_A,\quad \langle x \rangle_{\Delta u-\Delta d},\quad \ldots$$

## Transverse

$$\mathcal{O}_{\boxed{T}}^{\nu\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\boxed{\sigma}^{\nu\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$

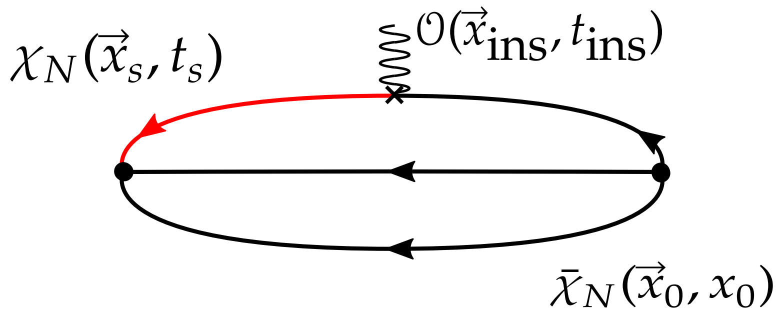


$$\langle 1 \rangle_{\delta u-\delta d} = g_T,\quad \langle x \rangle_{\delta u-\delta d},\quad \ldots$$

# Axial charge

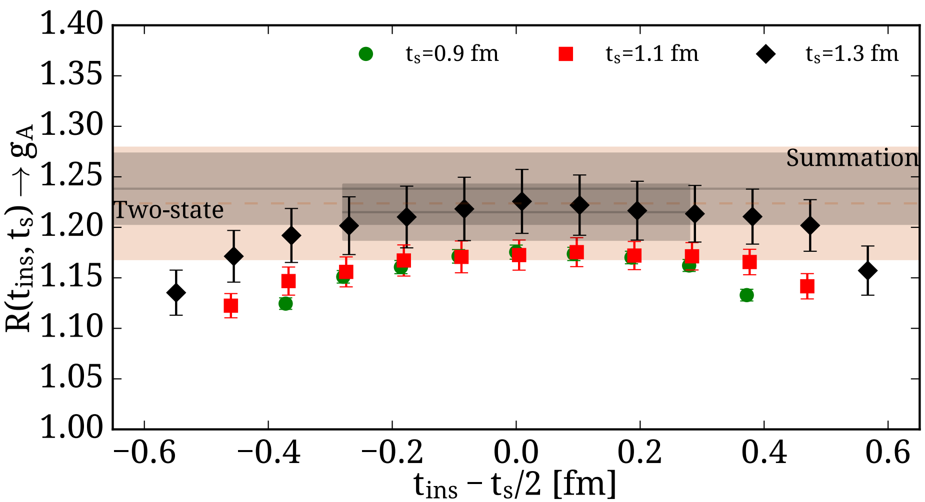
## Isovector matrix element:

$$\langle N|\bar{u}\gamma_5\gamma_k u - \bar{d}\gamma_5\gamma_k d|N\rangle \rightarrow g_A$$




Need simultaneously:

$$t_{\rm ins} \gg, \quad t_s - t_{\rm ins} \gg$$




Direct fit to single $t_s$

Fit to multiple $t_s$ including an excited state

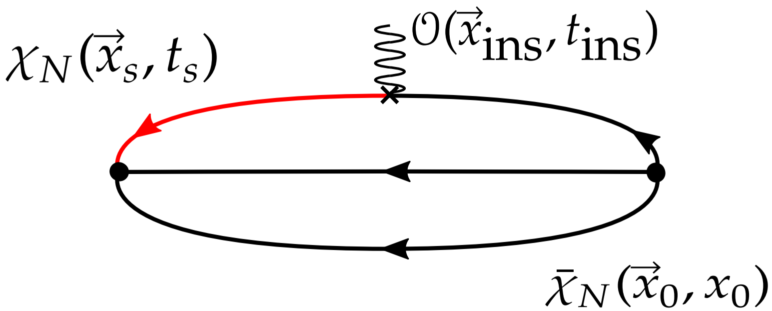
Fit to sum over $t_{ins}$ as function of $t_s$

$m_\pi$=135 MeV, Twisted Mass: Phys.Rev. **D92** (2015) 114513

# Axial charge

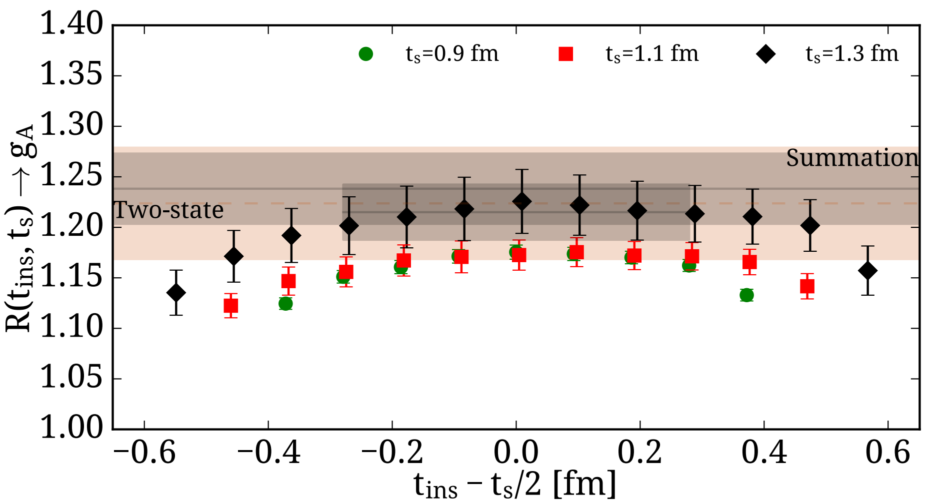
Isovector matrix element:

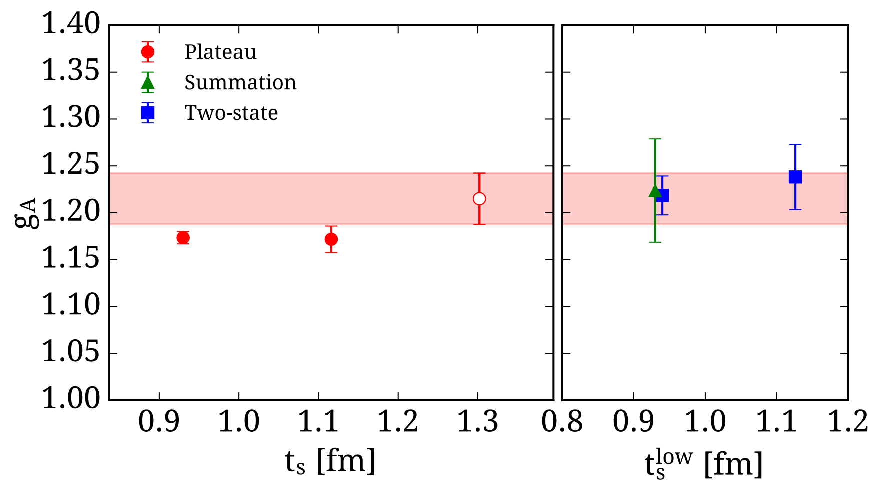
$$\langle N|\bar{u}\gamma_5\gamma_k u - \bar{d}\gamma_5\gamma_k d|N\rangle \to g_A$$




Need simultaneously:

$$t_{\text{ins}} \gg, \quad t_{\text{s}} - t_{\text{ins}} \gg$$

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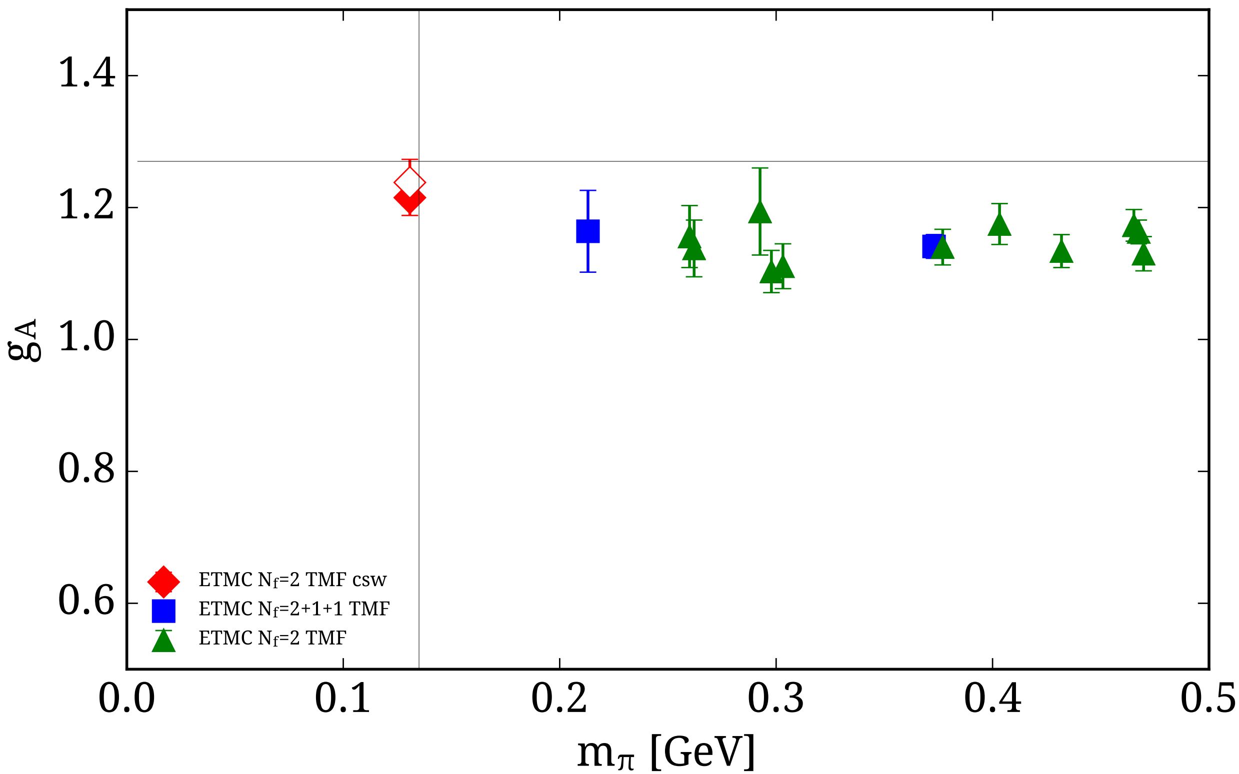
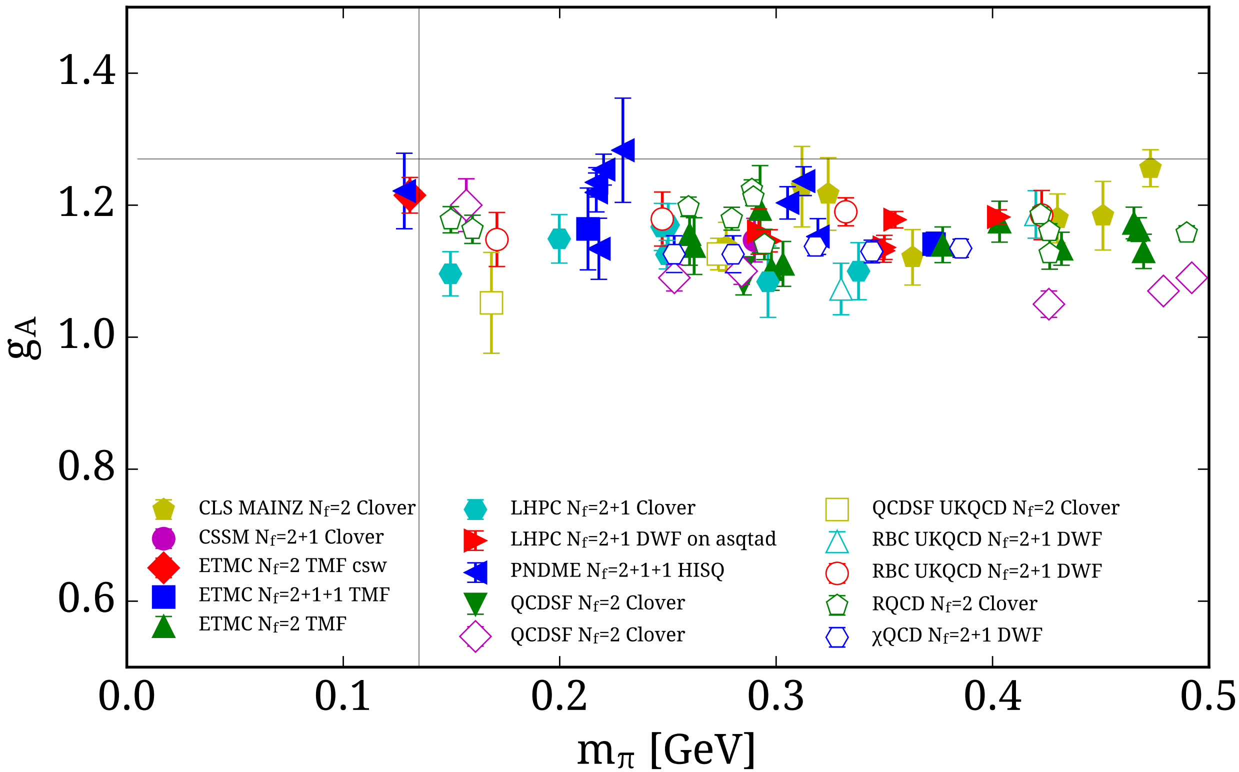



**Axial charge**

- Agreement towards experiment
- Simulations very close to or at the physical quark mass
- At physical point: statistics of ~$10^4$, up to 1.3 fm separation

# Axial charge

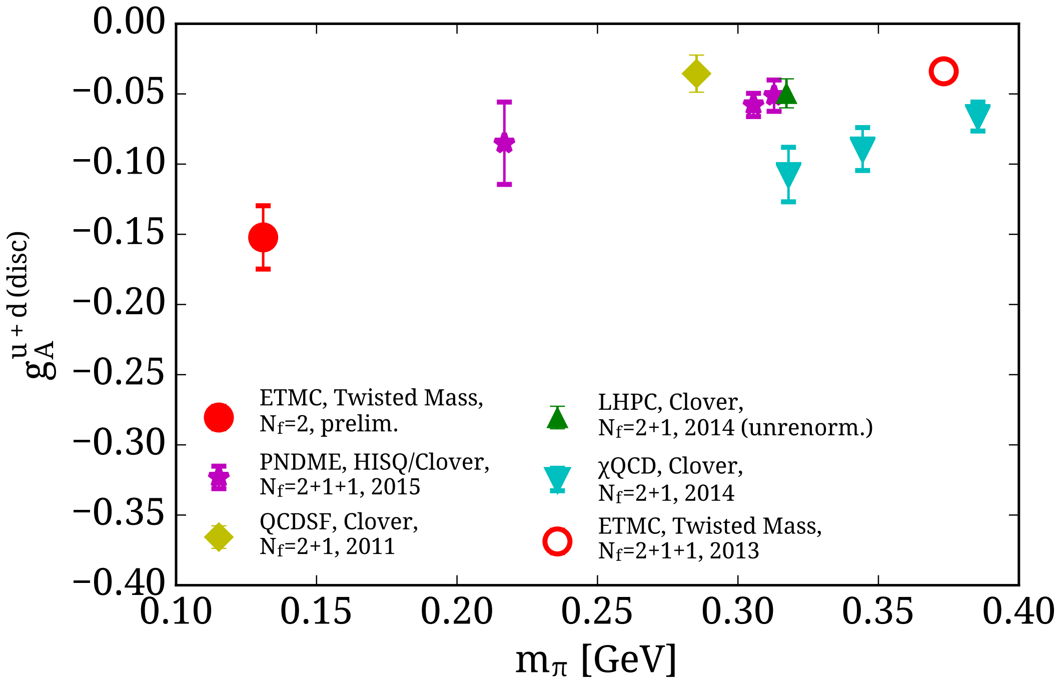
$$\langle N|\bar{u}\gamma_5\gamma_k u - \bar{d}\gamma_5\gamma_k d|N\rangle \rightarrow g_A$$

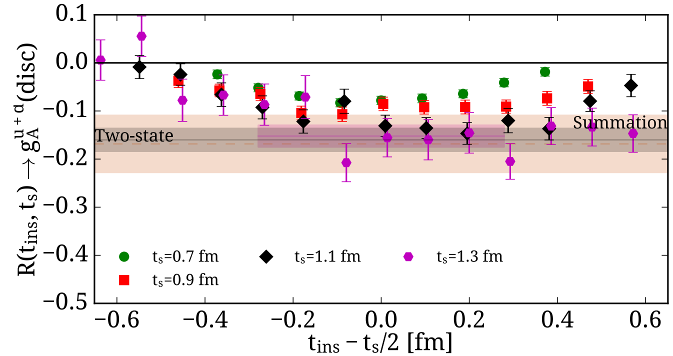


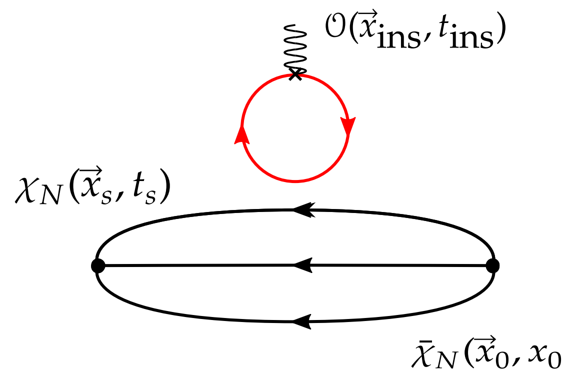

**Axial charge**

- Agreement towards experiment
- Simulations very close to or at the physical quark mass
- At physical point: statistics of ~$10^4$, up to 1.3 fm separation

$$\bar{u}\gamma_5\gamma_\mu u + \bar{d}\gamma_5\gamma_\mu d$$



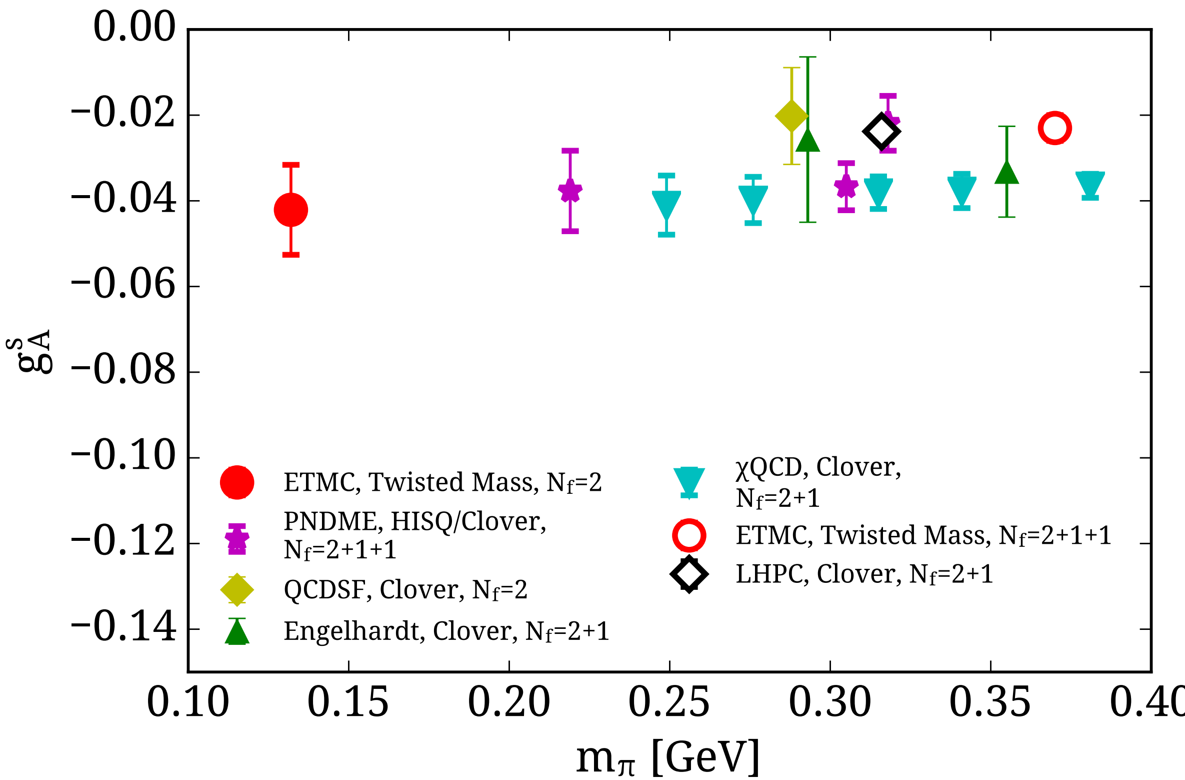



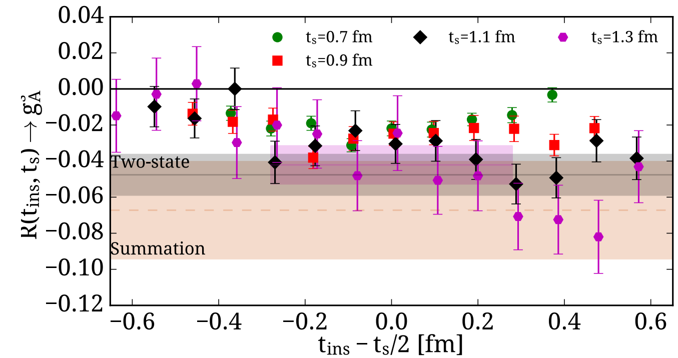



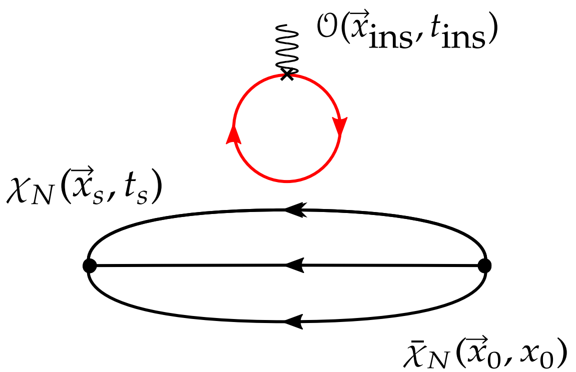

**Axial charge — light disconnected**

- Required for individual u- and d- contributions
- Requires dedicated calculations for “disconnected quark loop”
- Large statistical fluctuations in correlation functions
- $>10^5$ statistics at all available separations
- Sign is negative: brings connected result down
- About 10% of connected value

$\bar{s}\gamma_5\gamma_\mu s$




**Axial charge — strange contribution**

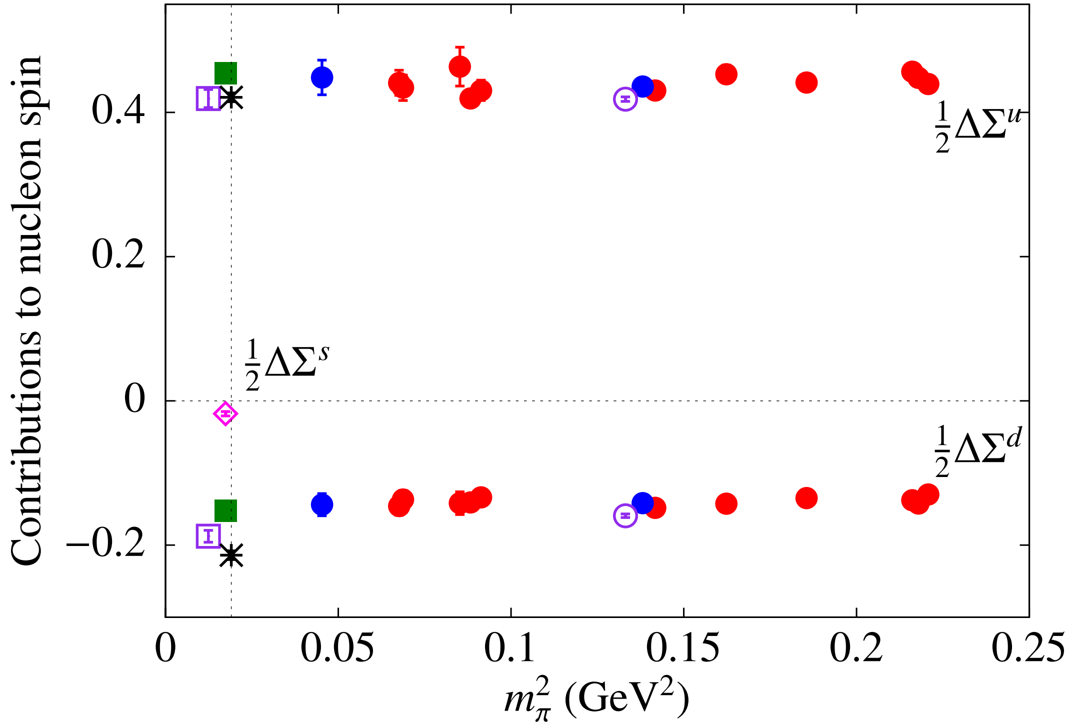
- Contribution exclusively by disconnected quark loop
- Results also at physical light quark masses
- Weak dependence on light quark mass

# Contributions to nucleon spin

$\Delta\Sigma^q = g_A^q$

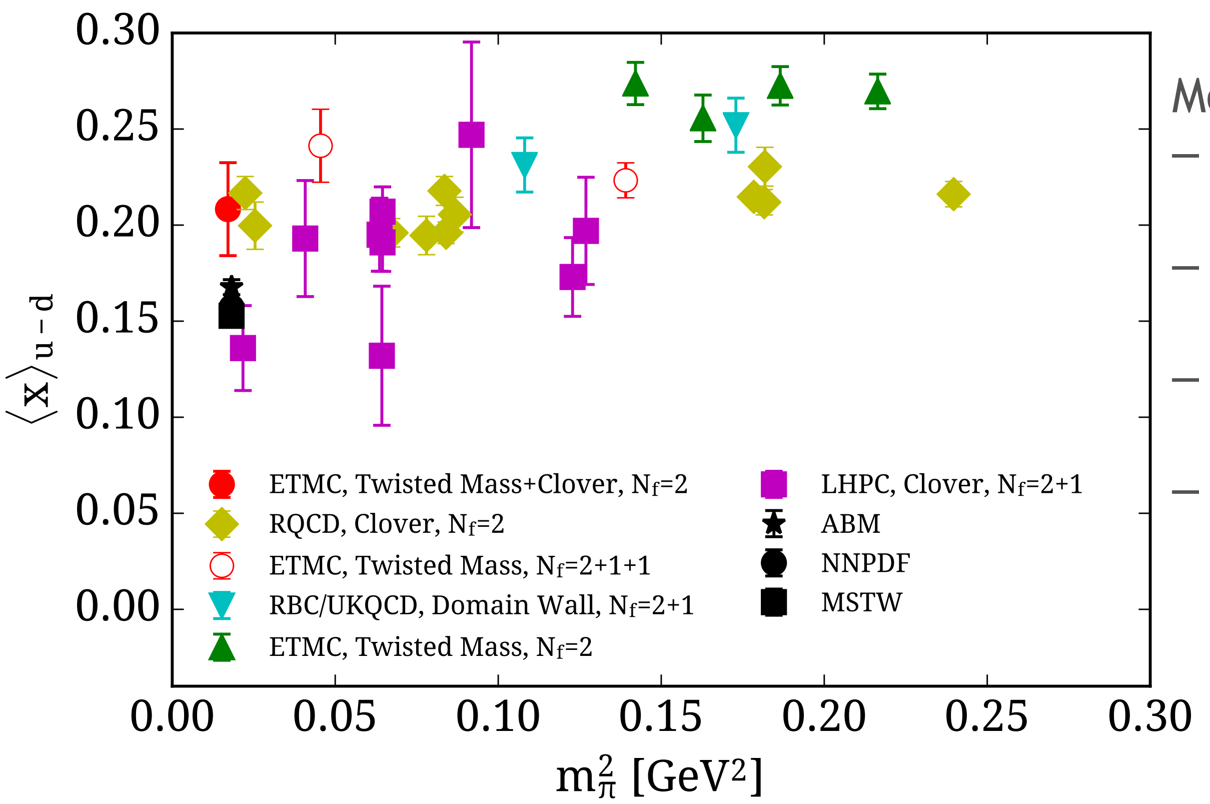
Axial charge — intrinsic quark spin contributions to nucleon spin

- Negative down-quark contribution
- Small strange-quark contribution
- Including disconnected contributions (open symbols) — better agreement with experiment (asterisks)



# Momentum fraction

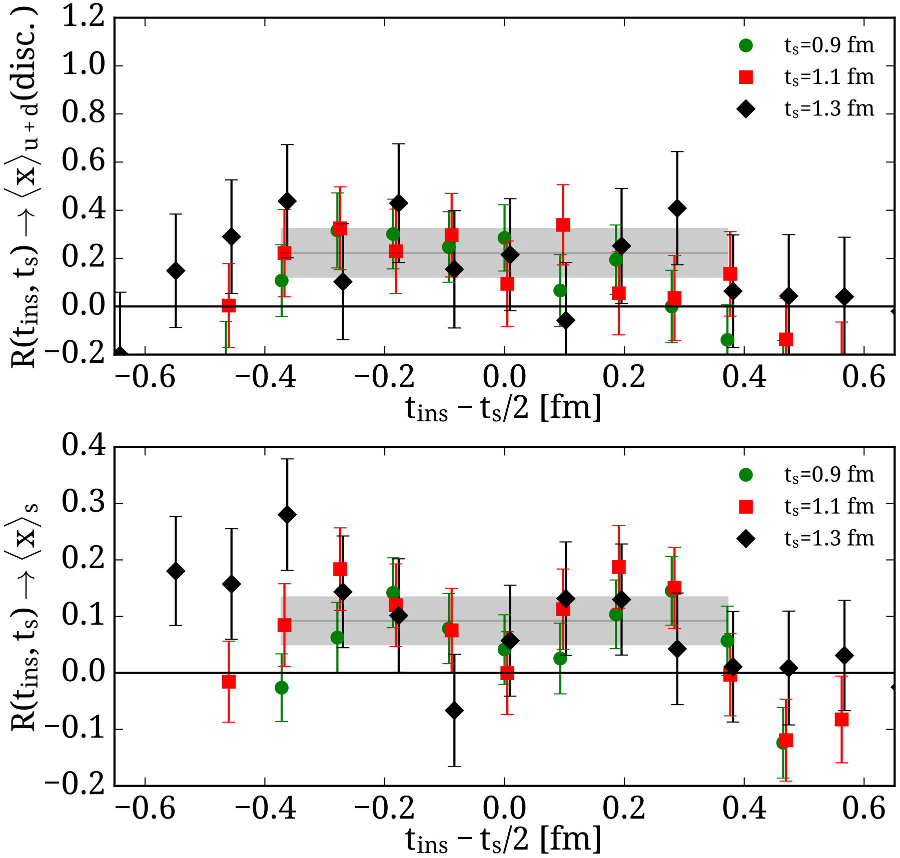
$$\mathcal{O}_V^{\mu\nu} = \bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi \rightarrow \langle x \rangle_q$$

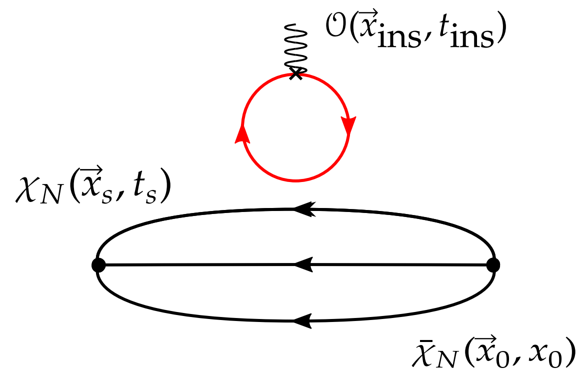



Momentum fraction

- General agreement between lattice actions
- More recent results at the physical point
- Pion mass dependence towards experimental values
- Need large separations ~1.5 fm

# Momentum fraction




Disconnected contribution

- Needed for isoscalar
- $O(10^5)$ statistics to obtain statistically non-zero value
- Needed for individual quark-contributions
- To obtain spin-decomposition of nucleon

# Spin decomposition

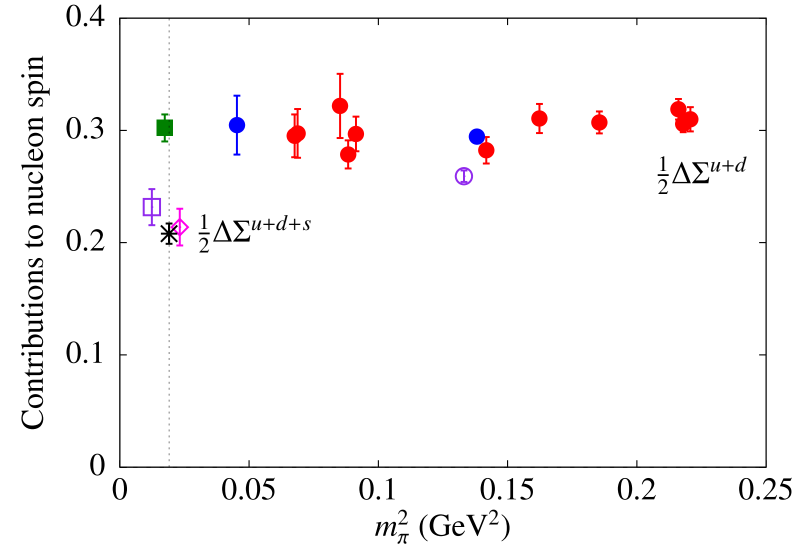
Ji's spin sum: $$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta\Sigma^q + L^q\right) + J^G$$

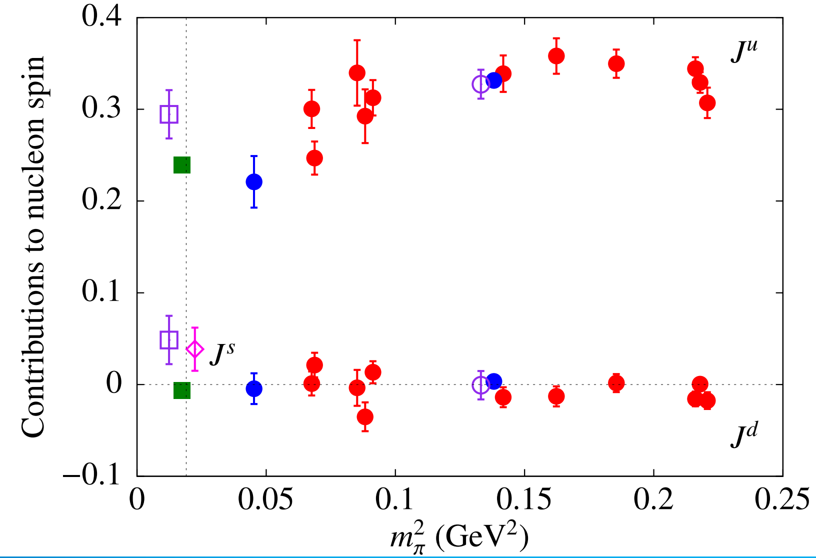
$$J^q = \frac{1}{2}\Delta\Sigma^q + L^q = \frac{1}{2}[A_{20}^q(0) + B_{20}^q(0)]$$

$$\Delta\Sigma^q = g_A^q$$

## Nucleon spin

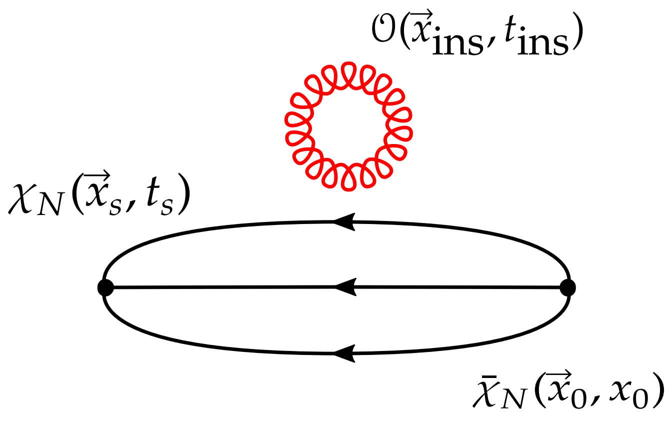
- Total $J$ spin contributions from unpolarised matrix element $A_{20}$, $B_{20}$.
- Intrinsic spin contributions from axial matrix element $\Delta\Sigma$
- Orbital angular momentum of quarks from difference
- $J^{u+d+s}$ = 0.398(60), implies $J^G$ = 0.102(60)



# Gluon moment

Ji's spin sum: $$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta\Sigma^q + L^q\right) + J^G$$



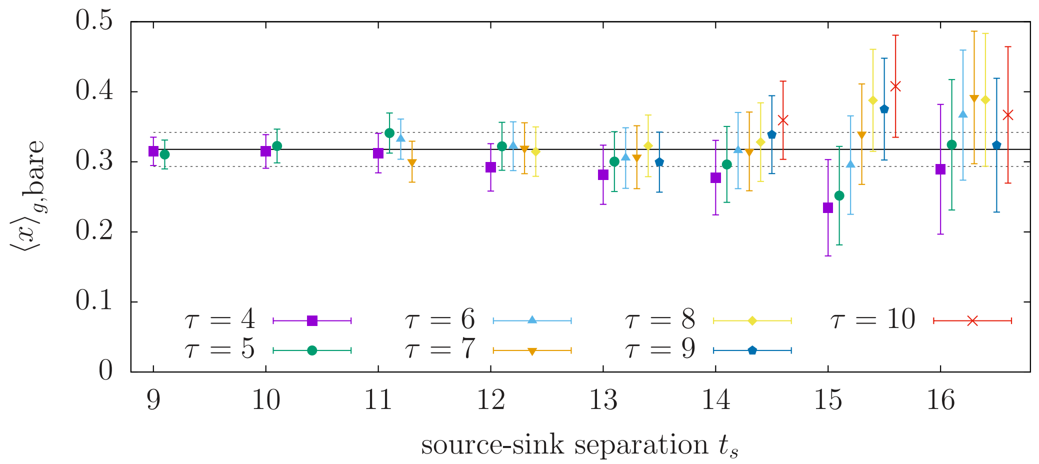

$$J^G = \frac{1}{2}[A_{20}^G(0) + B_{20}^G(0)]$$

$$\mathcal{O} = \frac{2}{9}\frac{\beta}{a^4}\left[\sum_i \Re(P_{i4}) - \sum_{i<j}\Re(P_{ij})\right]$$

Renormalisation

- Mixing with quark operator

$$\begin{pmatrix} J_q \\ J_G \end{pmatrix} = \begin{pmatrix} Z_{qq} & Z_{qG} \\ Z_{Gq} & Z_{GG} \end{pmatrix} \begin{pmatrix} J_q^{\text{bare}} \\ J_G^{\text{bare}} \end{pmatrix}$$



# Spin decomposition

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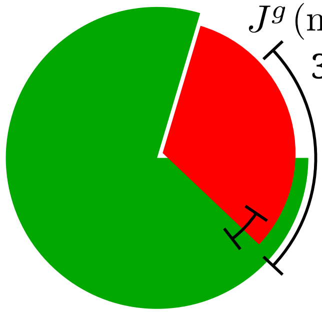
$$J^q = \frac{1}{2}\Delta\Sigma^q + L^q = \frac{1}{2}[A^q_{20}(0) + B^q_{20}(0)]$$

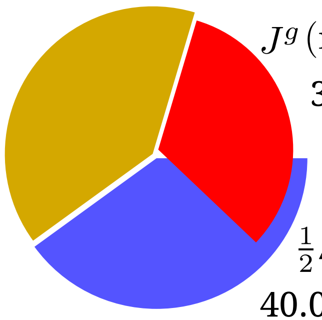
$$\Delta\Sigma^q = g^q_A$$

$$J^G = \frac{1}{2}[A^G_{20}(0) + B^G_{20}(0)]$$




## Nucleon spin

- $J^G$ = 0.160(12) from direct calculation PoS(DIS2016) [arXiv:1609.00253v1]
- $J^{u+d+s+G}$ = 0.541(59)



# Summary and outlook

★**Lattice QCD in new era**

- Physical pion mass simulations from a number of collaborations
- Other systematic uncertainties coming under control

★**Nucleon spin**

- Axial charge and momentum fraction at the physical point
- Results for disconnected fermion loops thanks to improved methods, order of magnitude more statistics
- New gluon moment calculations reveal gluon contribution to spin

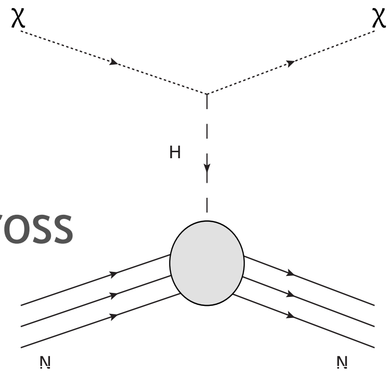
★**What to expect**

- Thorough estimation of systematic errors: continuum and infinite volume limits
- $B_{20}$ calculation for disconnected and gluon
- Longer term: effects of isospin breaking and electromagnetic effects

# Backup slides

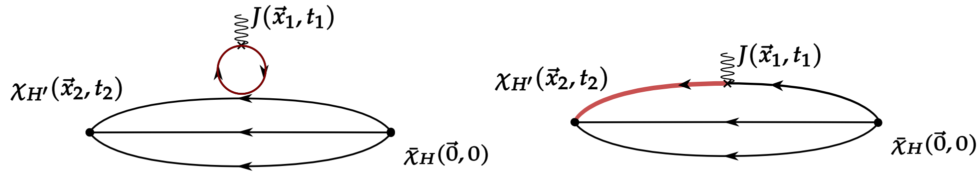
# Nucleon sigma – terms

- Pion nucleon σ-term: $\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$
- Strange σ-term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
- Enter super-symmetric candidate particle scattering cross sections with nucleon (e.g. neutralino through Higgs)

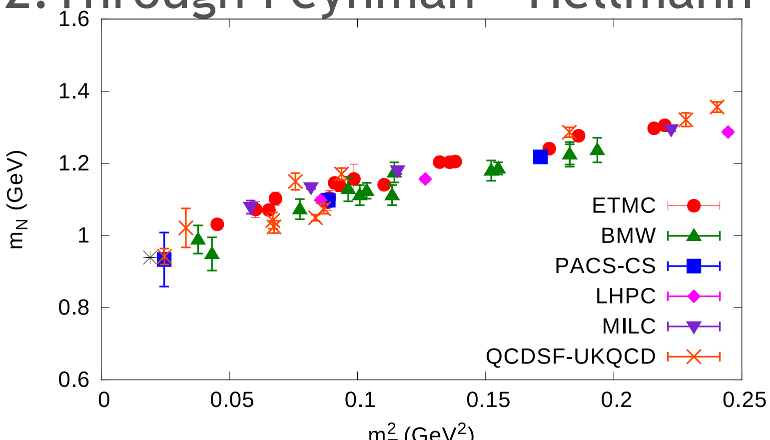



1. Direct calculation of matrix elements

Involves disconnected contributions

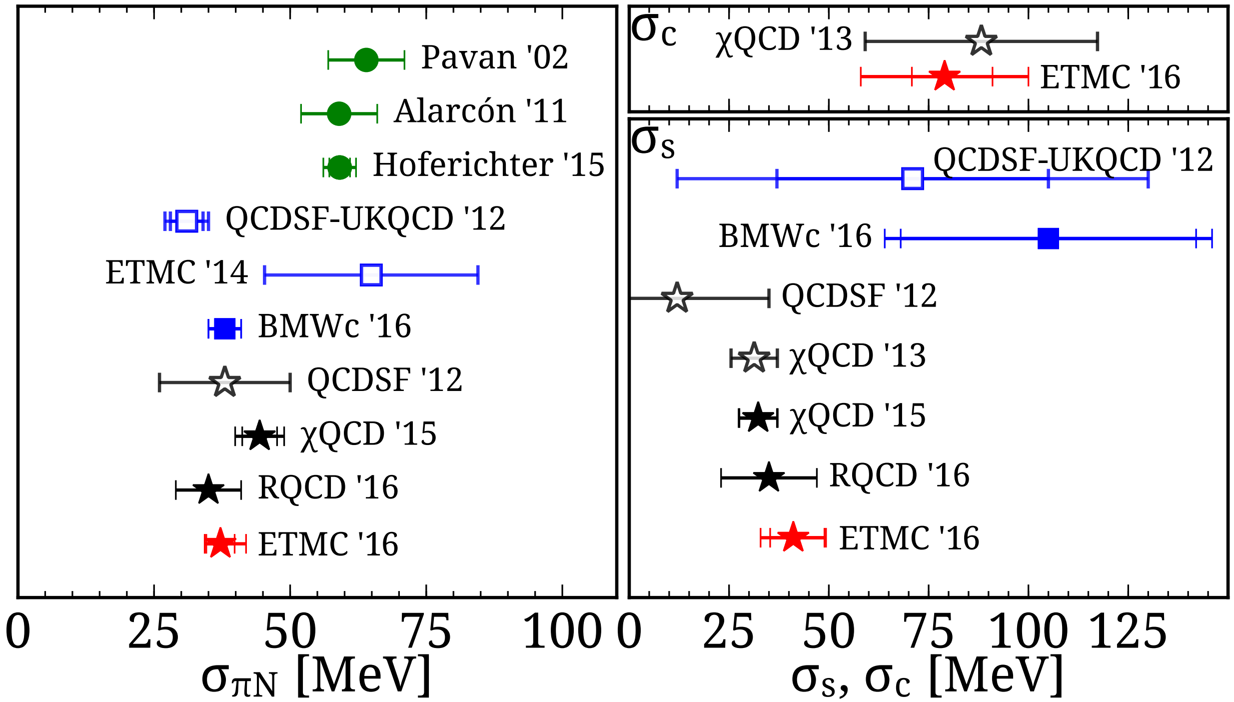



2. Through Feynman - Hellmann theorem: $\sigma_{\pi N} = m_{ud}\dfrac{\partial m_N}{\partial m_{ud}}$ $\sigma_s = m_s\dfrac{\partial m_N}{\partial m_s}$




- Reliance on effective theories for dependence on $m_\pi$
- Weak dependence on $m_s$

# Nucleon sigma — terms



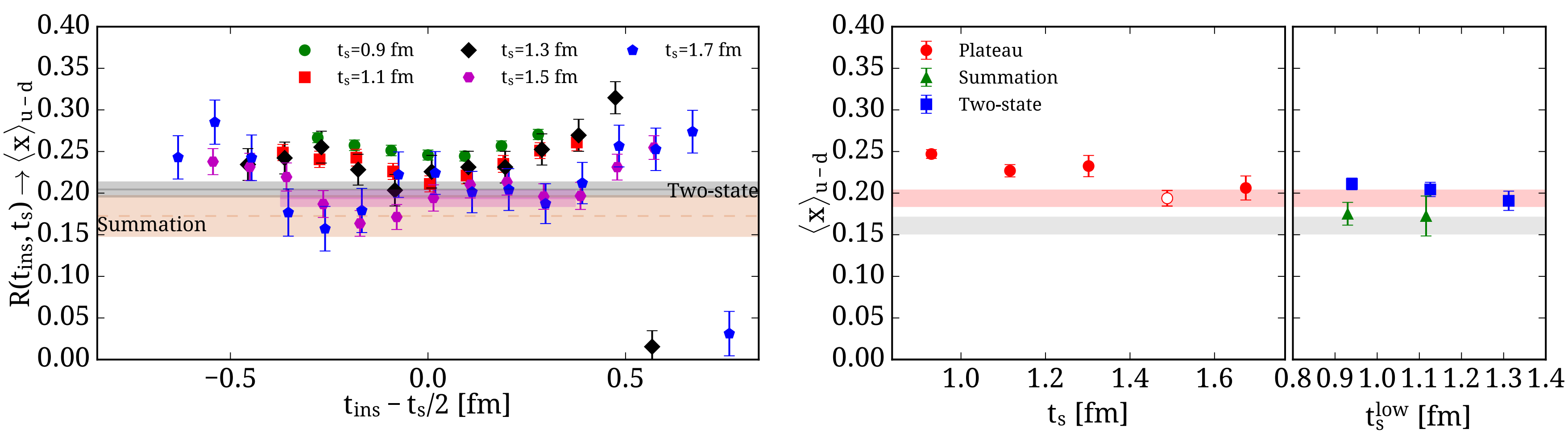

Phys. Rev. Lett. **116** (2016) 252001

- Recent results using direct matrix element evaluation [★]
- Large errors in FH-method [■](especially strange) due to sensitivity to quark mass
- Compare to phenomenology [●]

Momentum fraction

– Large sink-source separation

– >50000 statistics



# Convergence of strange and light disconnected axial charge